

## AN ANALYTICAL SOLUTION FOR YAW MANEUVER OPTIMIZATION ON THE INTERNATIONAL SPACE STATION AND OTHER ORBITING SPACE VEHICLES

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This paper presents a new method for optimizing yaw maneuvers, which are the most common large maneuvers on the International Space Station (ISS). The goal of the maneuver optimization is to find a maneuver trajectory with minimal torques acting on the vehicle during the maneuver. Therefore, the thruster firings necessary to perform the maneuver are minimized. Reduction of thruster firings saves propellant and decreases structural loads and contamination of the vehicle critical elements, thus saving the service life of the thrusters and the vehicle itself. Equations describing the pitch and roll motion needed to counteract the major torques during a yaw maneuver are obtained. Also, a yaw rate profile is suggested. In the obtained optimized case, the torques are significantly reduced. The proposed approximate analytical solution does not require extensive computer resources and, therefore, can be implemented using software onboard the ISS. As a result, the maneuver execution will be automatic. This is one of the major benefits of the simplified solution presented in this paper with respect to existing computational approaches. The suggested maneuver optimization method can be used not only for the ISS, but for other space vehicles as well.

## I. INTRODUCTION

ISS rotations about its center of mass are the standard operations of the space station. These operations are often needed for visiting vehicle docking and undocking events, orbit correction and debris avoidance maneuvers, Extravehicular Activities (EVA), and various experiments. For the ISS, yaw maneuvers are used most often. In this paper, an analytical solution for optimizing the ISS yaw maneuvers is suggested. When maneuver optimization is used, large maneuvers, which were performed using thrusters, could be performed either using Control Moment Gyroscopes (CMG), or with significantly reduced thruster firings.

One of the first approaches to the ISS maneuver optimization was discussed in the early stages of the ISS<sup>1</sup>. An ability to perform non-propulsive 180 degree yaw maneuvers of the ISS was first proven by the “Zero Propellant Maneuver” (ZPM) created by Draper Laboratory (USA) in 2006<sup>2, 3, 4</sup>. The optimal control problem was

solved using the Legendre pseudospectral method<sup>5</sup>. Each ZPM maneuver created by Draper Laboratory is unique, and can only be calculated on the ground since significant computer resources are needed for calculations. About 100-200 commands have to be sent onboard to execute the ZPM, making the operation rather complicated. Similar to ZPMs are Optimal Propellant Maneuvers (OPM)<sup>6</sup>. Maneuver duration for OPMs is less than for ZPMs, but OPMs cannot be performed without thruster firings. However, propellant consumption for OPMs is very low.

The goal of this research was to find a simplified maneuver optimization solution which, in contrast to the Draper Laboratory method, does not require a lot of computer resources. Therefore, the maneuver can be executed by the onboard software, thus significantly simplifying operations. It also allows the maneuver to be performed with no communication with the ground. The analytical solution can be examined for different parameter configurations, and it is

possible to estimate the effectiveness of maneuver optimization for different variations of the ISS mass properties. Also, the highest possible rate of the optimal maneuver for a specific mass property configuration can be found.

## II. MODEL

The following assumptions are used in this paper:

1. The ISS is a rigid body.
2. The aerodynamic torques are not taken into account.

The second assumption can be used because the aerodynamic torques are in the order of several N-m, and this is about 50 - 100 times less than the ISS torques which are intended to be reduced with the proposed method. Simulations using the ISS flight software showed that these assumptions are sufficient to obtain a maneuver solution which can significantly reduce the torques.

## III. COORDINATE SYSTEMS

Introducing the following coordinate systems:

1. LVLH system (Local Vertical. Local Horizontal) ( O X,Y,Z)
  - The origin O is in the ISS center of mass;
  - The +X axis of LVLH system is in the direction of the vehicle velocity vector;
  - The +Z axis of LVLH system is directed towards the center of the Earth;
  - The +Y axis of LVLH system is perpendicular to the orbit plane and completes the right handed coordinate system.
2. The J2000 inertial coordinate system
  - The origin O<sub>J2000</sub> is in the center of the Earth;
  - The +X<sub>J2000</sub> axis points towards the mean vernal equinox at noon of January 1, 2000 and lies in the equatorial plane;
  - The +Z<sub>J2000</sub> axis is perpendicular to the equatorial plane and points towards the North pole along the Earth's mean rotational axis;

- The +Y<sub>J2000</sub> axis completes the right-handed coordinate system.

3. The inertial coordinate system with the origin in the ISS center of mass and with the axis parallel to the axis of J2000. (O X<sub>I</sub>Y<sub>I</sub>,Z<sub>I</sub>)
4. The ISS body coordinate system (O x, y, z). The axes of this system are aligned with the ISS principal axes.

## IV. THE ISS EQUATIONS OF ROTATION

The ISS equation of rotation around its center of mass are described in yaw, pitch, and roll Euler angle sequence.

The equations of the ISS rotation as a rigid body have a well-known expression:

$$\begin{aligned} A\dot{\omega}_x + (C - B)\omega_y\omega_z &= T_x \\ B\dot{\omega}_y + (A - C)\omega_z\omega_x &= T_y \\ C\dot{\omega}_z + (B - A)\omega_x\omega_y &= T_z \end{aligned} \quad [1]$$

Where:

- A, B, C are the ISS principal moments of inertia with respect to axes Ox, Oy, and Oz correspondingly.
- T<sub>x</sub>, T<sub>y</sub>, T<sub>z</sub> are the components of external moments of force acting on the station with respect to axis Ox, Oy, and Oz correspondingly.
- $\omega$  is angular rate of the ISS with respect to the ISS inertial system.
- $\omega_x, \omega_y, \omega_z$  are the projections of the angular rate  $\omega$  on the ISS principal axes.

For the pure yaw maneuver the roll and pitch angles are zero. For yaw maneuver optimization we will look for the cases when the roll and pitch angles are small. The physical reasons for using this approximation will be discussed later.

Expressing [1] in Euler angles, and assuming a small angle approximation for roll ( $\gamma$ ) and pitch ( $\beta$ ) angles, the equations [1] will have the form:

$$\begin{aligned}
& A(\ddot{\gamma} - \ddot{\alpha}\beta) + (C - B - A)\dot{\alpha}\dot{\beta} \\
& + (C - B)(\dot{\alpha}^2\gamma + n^2\gamma\sin^2\alpha \\
& \quad + n^2\beta\cos\alpha\sin\alpha) - \\
& - (C - B + A)n\dot{\alpha}\cos\alpha - 4n^2(C - B)\gamma \\
& = T_x
\end{aligned}$$

$$\begin{aligned}
& B(\ddot{\beta} + \ddot{\alpha}\gamma) - (C - B - A)\dot{\alpha}\dot{\gamma} \\
& + (C - A)(\dot{\alpha}^2\beta + n^2\beta\cos^2\alpha \\
& \quad + n^2\gamma\cos\alpha\sin\alpha) + \\
& + (C + B - A)n\dot{\alpha}\sin\alpha - 4n^2(C - A)\beta \\
& = T_y
\end{aligned} \tag{2}$$

$$\begin{aligned}
& C\ddot{\alpha} + (C - B + A)n\cos\alpha(\dot{\gamma} - \dot{\alpha}\beta) \\
& - (C + B - A)n\sin\alpha(\dot{\beta} + \dot{\alpha}\gamma) + \\
& + (B - A)n^2\cos\alpha\sin\alpha = T_z
\end{aligned}$$

Where

$\alpha$  is the yaw angle;

$\beta$  is the pitch angle;

$\gamma$  is the roll angle;

$n$  is the orbital rate.

$T_x, T_y, T_z$  include only the thruster firing torques.

Using a small angle approximation, the gravity torques in equations [2] are expressed as:

$$\begin{aligned}
T_{gravity_x} &= 3n^2(C - B)\gamma \\
T_{gravity_y} &= 3n^2(C - A)\beta \\
T_{gravity_z} &= 0
\end{aligned} \tag{3}$$

## V. FORMULATING THE PROBLEM

The task is to perform the 180 degree rotation around the Z axis of LVLH system minimizing the magnitudes of control torques.

Assuming the initial conditions:  $\alpha = 0, \beta = 0, \gamma = 0, \dot{\alpha} = 0, \dot{\beta} = 0, \dot{\gamma} = 0$ .

The required final conditions:  $\alpha = 180, \beta = 0, \gamma = 0, \dot{\alpha} = 0, \dot{\beta} = 0, \dot{\gamma} = 0$ .

The goal is to find  $\alpha(t), \beta(t),$  and  $\gamma(t)$  which will bring the ISS from the initial to the final

position with the minimized control torques (minimized thruster firings or minimized increase of CMG momentum).

We will start with selecting  $\alpha(t)$  profile and then will obtain the corresponding  $\beta(t)$  and  $\gamma(t)$  profiles.

## VI. YAW RATE PROFILE SELECTION

To perform a 180 degree yaw maneuver on CMGs the yaw rate and acceleration should be low enough so that the yaw torque is as low as the CMGs are able to control. This requirement sets the limitation for how fast the maneuvers on CMGs can be performed.

For a CMG maneuver, the optimal yaw rate profile should also help to avoid high accelerations since the CMGs cannot handle high torques. To satisfy these requirements, a bell profile, which is close to a triangle, was selected. The maximum acceleration is the smallest for such a profile. The profile should also provide zero rate and acceleration at the start and the end of the maneuver.

Different functions can be used to describe this type of a profile. The 4<sup>th</sup> order polynomial, which is currently used as an option in the ISS onboard software, provides a good description of the desired profile.

$$\dot{\alpha}(t) = K(1 - \tau)^2\tau^2, \text{ where } \tau = \frac{t}{T} \tag{4}$$

Coefficient K depends on the duration of the maneuver and the maneuver angle-to-go.

The plot shown in red in Figure 1 illustrates the profile described by equation [4].

*Legend:*

Red line – yaw rate profile described by equation [4],

Blue line – yaw rate profile modification example.

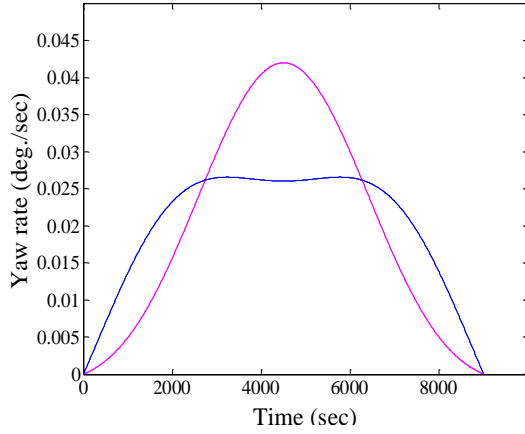


Figure 1. Yaw rate profiles

Note that the yaw rate profile can be modified for specific requirements, scenarios, mass properties or thruster configurations. For example, the yaw rate profile can be changed to reduce roll and pitch torques while increasing the yaw torque during maneuver. This may be useful when roll or pitch control is not as effective as yaw or for other considerations such as loads, contamination, or thruster issues. The plot shown in blue in Figure 1 illustrates a possible yaw profile modification.

However, in general case, the profile suggested by equation [4] provides good optimization results. After the  $\alpha(t)$  profile is selected, the next step is to obtain the corresponding optimal  $\beta(t)$  and  $\gamma(t)$  profiles.

## VII. YAW MANEUVER TORQUES

Let us first consider the 180 degree pure yaw maneuver, where roll and pitch angles remain zero. This maneuver is performed using thrusters.

For such a maneuver the torques in roll and pitch can be obtained using equations [2]:

$$\begin{aligned} -(C - B + A)n\dot{\alpha}\cos\alpha &= T_x \\ (C + B - A)n\dot{\alpha}\sin\alpha &= T_y \end{aligned} \quad [5]$$

The magnitude of these torques depends on the mass properties, yaw maneuver rate  $\dot{\alpha}$ , and the orbital rate. For the ISS mass properties and for

the common ISS maneuver durations of about 5400 seconds or less (or even for significantly longer maneuvers) these torques are large with respect to the CMG capabilities. Therefore, pure yaw maneuvers cannot be performed without thrusters.

As an example, Figure 3 below compares the onboard roll torque telemetry for a pure 180 degree ISS yaw maneuver (blue line) and the torque in roll calculated per equation [5] (red line).

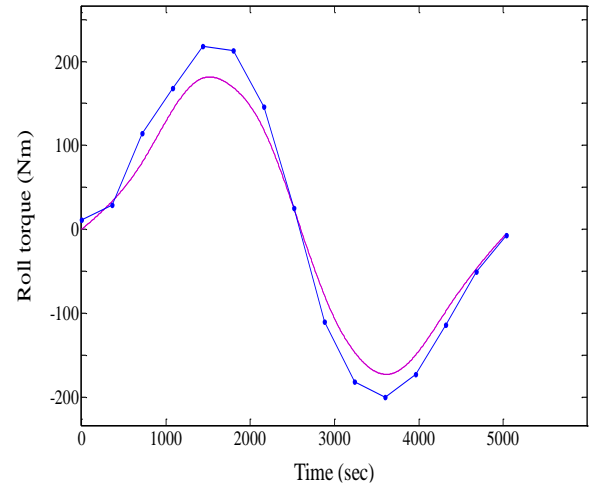


Figure 2. Roll torque during a 180 degree pure yaw maneuver.

The telemetry plot profile in Figure 2 is close to the torque calculated per equation [5]. Some difference may be due to several factors: variation in mass properties, atmospheric drag, and an error in attitude knowledge and control system performance.

It can also be seen from this plot that the torque is rather large, about 200 N·m at maximum, which is much more than the ISS CMGs can handle without desaturation. The torque in pitch during the pure yaw maneuver has a similar large magnitude. Therefore, the goal of a maneuver optimization is to reduce these torques.

Gravity torques can be used to solve the problem by compensating for the torques described by equation [5]. This compensation is the essence of

the suggested method of reducing torques during yaw maneuvers.

### VIII. MANEUVER OPTIMIZATION LIMITATIONS

The proposed maneuver optimization is not possible for all ranges of mass properties of any given space vehicle. The gravity torques have to be big enough to compensate for the pure yaw maneuver torques. Note that in an extreme case when  $C = A$ , or  $C = B$  the proposed maneuver optimization is impossible since per equations [3] the corresponding gravity torques are zero.

In general, the optimization is more effective with larger gravity gradients in pitch and roll, and lower maneuver rate.

In this work it is assumed that the gravity gradients are large enough to create the required gravity torques even with the small roll and pitch angles. In this case a small roll and pitch angle approximation can be used, and the torques necessary to create the roll and pitch rotations are small compared to the pure yaw maneuver torques, which are to be compensated.

Simple computations, which compare torques before and after optimization, can determine if the suggested maneuver optimization method is possible for each specific vehicle. Calculations prove that this method is applicable for the ISS.

### IX. FIRST APPROXIMATION SOLUTION

Considering the case of large gravity gradients in pitch and roll, and small pitch and roll angles, rates, and accelerations, the following simplified solution can be suggested as a first approximation for roll and pitch profiles:

$$\begin{aligned}\gamma_0 &= \lambda \dot{\alpha} \cos \alpha, \\ \beta_0 &= \mu \dot{\alpha} \sin \alpha\end{aligned}\tag{6}$$

Where

$$\lambda = -\frac{(C-B+A)}{4n(C-B)}, \quad \mu = \frac{(C-A+B)}{4n(C-A)}\tag{7}$$

For the possible range of the ISS mass properties and for the commonly used range of the ISS maneuver rates it was shown, by substituting equations [4], [6], [7] in the system [1], that solution [6], [7] can significantly reduce the roll and pitch control torques during ISS yaw maneuvers. The examples of this torque reduction are shown in the next section.

### IX. I. Torque Reduction Examples

Figures 3, 4, and 5 below provide an example of yaw maneuver torque reduction on the ISS when the roll and pitch profiles are defined by equations [6] and [7]. First, let us look at the plots shown in red and in blue in these figures. The plots in red illustrate the torques for the non-optimized pure yaw maneuver. The plots in blue illustrate the torques for an optimized maneuver described by solution [6], [7]. Comparing these torques, we see that the solution [6], [7] significantly reduces the maneuver torques. The maneuver duration in this example is 9000 seconds.

(Note that the torques for these plots and all the following plots are calculated using the full system of equations [1] without the small angle approximation).

*Legend:*

Red line – non-optimized pure yaw maneuver,  
Blue line – solution [6], [7],  
Black line – solution [8], [9].

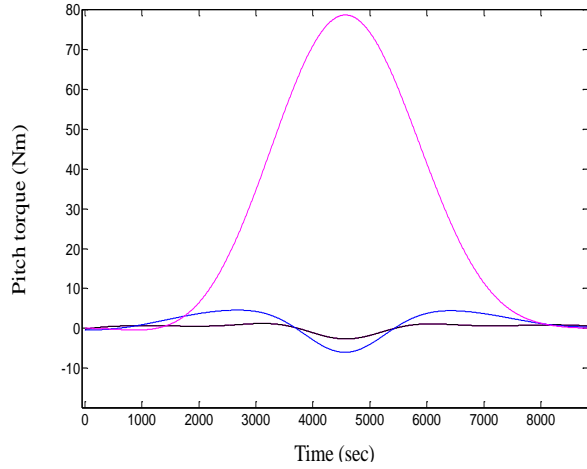


Figure 3. Pitch torque reduction for optimized 9000 second yaw maneuver.

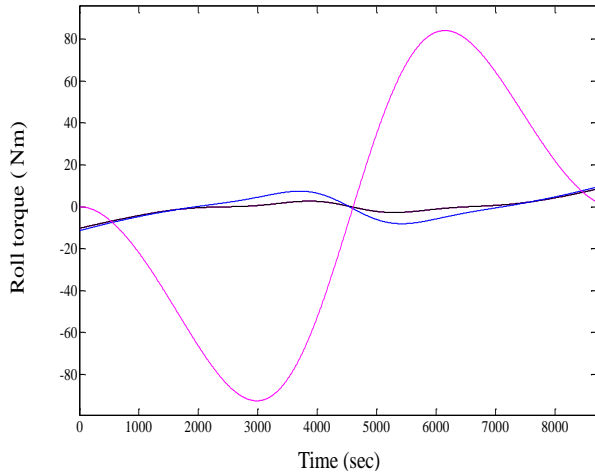


Figure 4. Roll torque reduction for optimized 9000 second yaw maneuver.

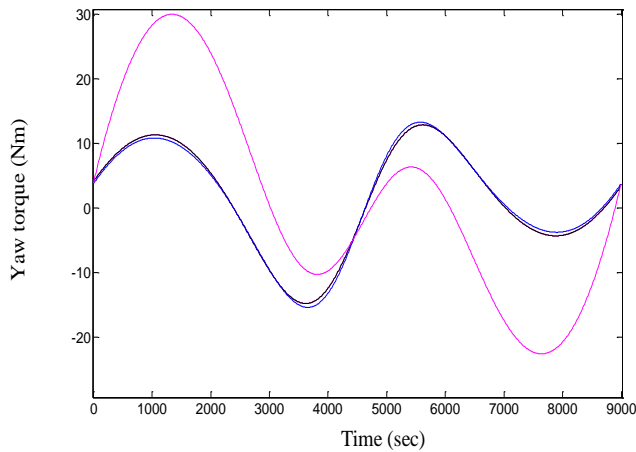


Figure 5. Yaw torque reduction for optimized 9000 second yaw maneuver.

While the first approximation solution [6], [7] considerably reduces torques in the above example, the torque reduction can be improved further by adding another term to the solution as a second approximation.

#### X. SECOND APPROXIMATION SOLUTION

$$\begin{aligned}\gamma &= \lambda \dot{\alpha} \cos \alpha + \lambda_1 \ddot{\alpha} \sin \alpha + \lambda_2 \ddot{\alpha} \cos \alpha \\ \beta &= \mu \dot{\alpha} \sin \alpha + \mu_1 \ddot{\alpha} \cos \alpha + \mu_2 \ddot{\alpha} \sin \alpha\end{aligned}\quad [8]$$

Where

$$\begin{aligned}\lambda &= -\frac{(C-B+A)}{4n(C-B)}, \quad \mu = \frac{(C-A+B)}{4n(C-A)} \\ \lambda_1 &= \frac{-3A\lambda - A\mu + (C-B-A)\mu}{4n^2(C-B)} \\ \mu_1 &= \frac{-3A\lambda - A\mu + (C-B-A)\mu}{4n^2(C-A)} \\ \lambda_2 &= \varepsilon_1 \frac{A\lambda}{4n^2(C-B)} \\ \mu_2 &= \varepsilon_2 \frac{B\mu}{4n^2(C-A)}\end{aligned}\quad [9]$$

This solution was obtained by substituting the first approximation solution [6], [7] into equations [2] and omitting the terms of the higher order of smallness. Note that for this solution the selected yaw rate profile should have  $\ddot{\alpha} = 0$  at the start and the end of the maneuver to match the initial and final conditions. For actual applications the term with  $\ddot{\alpha}$  may be small with respect to vehicle control system deviations.

Solution [8], [9] provides a good approximation for ISS yaw maneuver optimization. For other vehicles two additional terms, which are negligible for the ISS, can improve the accuracy of the approximation. The modified roll and pitch profiles  $\gamma_m$  and  $\beta_m$  with these additional terms are:

$$\begin{aligned}\gamma_m &= \gamma + \lambda_3 \dot{\alpha}^3 \cos \alpha + \lambda_4 \dot{\alpha} \cos \alpha \sin^2 \alpha \\ \beta_m &= \beta + \mu_3 \dot{\alpha}^3 \sin \alpha + \mu_4 \dot{\alpha} \sin \alpha \cos^2 \alpha\end{aligned}\quad [10]$$

Where

$\gamma$  and  $\beta$  are obtained from [8], [9],

$$\begin{aligned}\lambda_3 &= \frac{(C-B-A)(\lambda+\mu)}{4n^2(C-B)} \\ \mu_3 &= \frac{(C-B-A)(\lambda+\mu)}{4n^2(C-A)} \\ \lambda_4 &= \frac{(\lambda+\mu)}{4}, \quad \mu_4 = \frac{(\lambda+\mu)}{4}.\end{aligned}\quad [11]$$

Solution [8], [9] contains terms with parameters  $\varepsilon_1$  and  $\varepsilon_2$  which are not defined as functions of the moments of inertia. For the ISS, calculations showed that the effect of these parameters is small. For the low rate maneuvers (duration of about 9000 seconds or more)  $\varepsilon_1$  and  $\varepsilon_2$  can be set to be equal to 1. However, for the faster rate maneuvers (duration of about 5400 seconds) these parameters may have a more noticeable effect, and therefore, it may be beneficial to find their optimal values which can be done by comparing control torques.

An additional calculation is necessary to obtain the optimal values of  $\varepsilon_1$  and  $\varepsilon_2$  for different mass properties, but this calculation is not complicated since the torque depends weakly on  $\varepsilon_1$  and  $\varepsilon_2$ , and it is not necessary to have small steps for variation of these parameters. For the ISS, these values are:  $\varepsilon_1 = 0.1$ ,  $\varepsilon_2 = 0.6$ .

It should be noted that solution [8], [9] provides the torque reduction for only a certain range of mass properties and maneuver durations. However, calculations show that it can be successfully used for the maneuver optimization for the whole possible range of the ISS mass properties.

#### XI. ISS YAW MANEUVERS THAT CAN BE PERFORMED WITHOUT THRUSTERS

The plots shown in black in Figures 3, 4, and 5 above illustrate the torques for an optimized maneuver described by equations [8], [9]. Comparing these plots with the plots in blue we can see that the difference in torque reduction

between the first and second approximation solutions for the 9000 second maneuver is small. It is shown that the torques for the 180 degree yaw maneuvers with a duration of 9000 seconds can be reduced to a low level at which CMGs can control these torques. Therefore, these maneuvers may be performed without thruster firings. Such maneuvers are called the ZPMs, as discussed previously.

Figures 3, 4, and 5 demonstrate that the torques are reduced most with the second order approximation solution. However, the difference in torque reduction between the two approximations is 4 N·m at maximum, which is not a significant difference for the ISS control system.

In conclusion, for the maneuvers with a duration of 9000 seconds or more, both the first and second approximation solutions are significantly reducing the torques, and either solution can be used to reduce the torques to the level that CMGs can handle.

#### XII. ISS YAW MANEUVERS THAT CANNOT BE PERFORMED WITHOUT THRUSTERS

For faster maneuvers the effect of using different approximations is more significant.

ISS 180 degree yaw maneuvers with a duration of about 5400 seconds or less cannot be performed without thruster support. Even if these maneuvers are optimized, torques cannot be reduced to the level at which CMGs can control them. However, the propellant required for the optimized maneuver is significantly reduced. As mentioned previously, the maneuvers of this kind are called the OPMs.

Plots below show the torque reduction with different solutions for the 5400 second 180 degree yaw maneuver.

*Legend:*

Red line – non optimized pure yaw maneuver,  
Blue line – solution [6], [7],  
Black line – solution [8], [9].

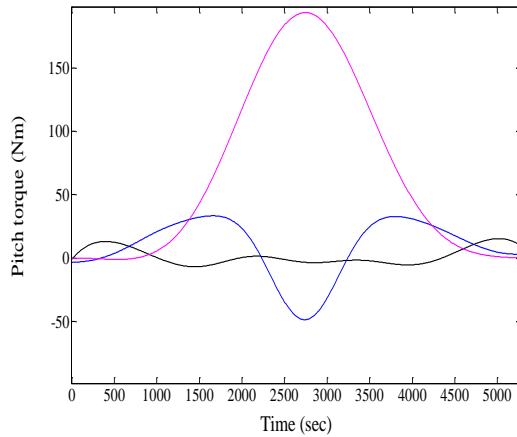


Figure 6. Pitch torque reduction for optimized 5400 second yaw maneuver.

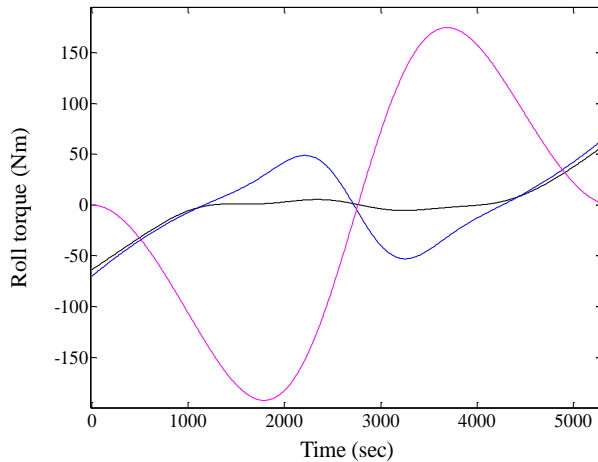


Figure 7. Roll torque reduction for optimized 5400 second yaw maneuver.

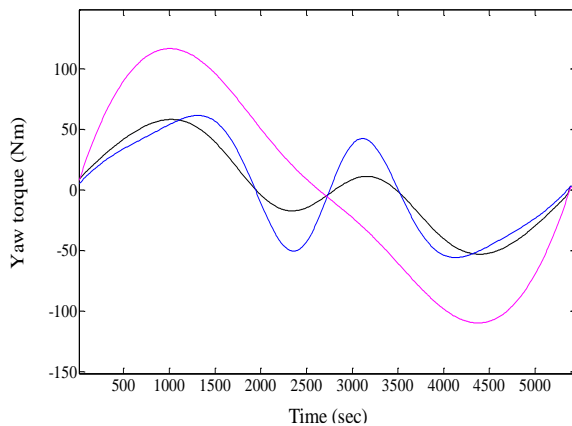


Figure 8. Yaw torque reduction for optimized 5400 second yaw maneuver.

The plots in Figures 6, 7, and 8 illustrate that the second approximation solution significantly reduces all the torques, and is noticeably better than the first approximation for the 5400 second maneuver.

The proposed optimization solution for the ISS yaw maneuvers is approximate, but simulations showed that it provides the maneuver performance similar to the existing computational solutions.

Section XIII below presents the comparison results of the suggested solution to the optimal maneuver calculated by Draper Laboratory using the computational approach.

### XIII. COMPARISON OF MANEUVER OPTIMIZATION SOLUTIONS

Calculations using the suggested method were done for a number of yaw maneuvers at different time periods starting from year 2006 to 2014. All the calculations showed the torque reduction for the maneuvers similar to the examples demonstrated above. The results were compared to the existing cases of Draper Laboratory calculations. The roll and pitch profiles and the torque reductions related to these profiles were compared. Since the maneuver optimization is more difficult to provide for the faster maneuvers, the comparison results for a faster maneuver are presented below.

1). Maneuver duration 5400 seconds.

Figures 9 and 10 provide comparison of the roll and pitch profiles.

*Legend:*

Blue line – analytical solution [8], [9],

Brown line – Draper Laboratory computational solution.



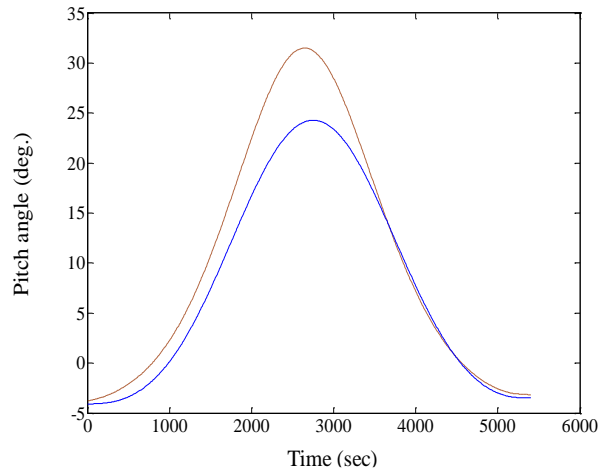


Figure 9. Pitch profile for optimized yaw maneuver.

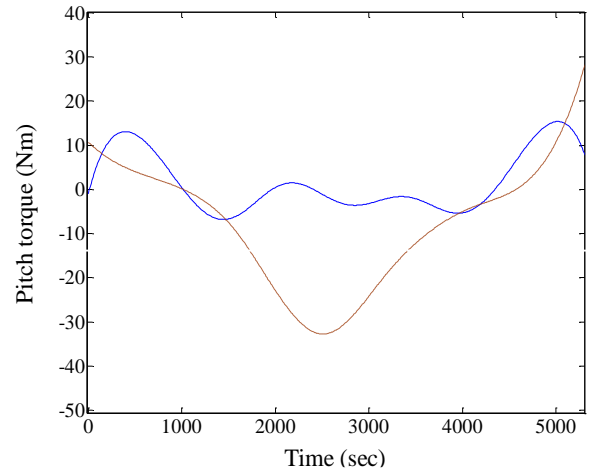


Figure 11. Pitch torque comparison for the 5400 second yaw maneuver.

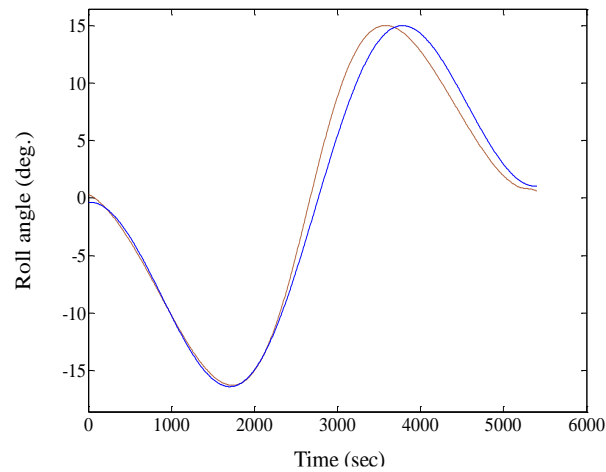


Figure 10. Roll profile for optimized yaw maneuver.

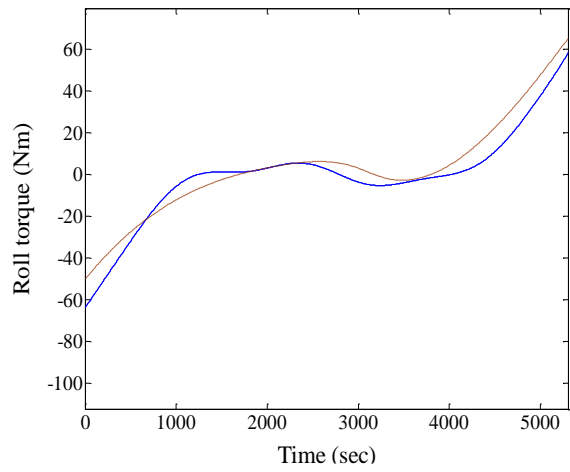


Figure 12. Roll torque comparison for the 5400 second yaw maneuver.

It can be seen from the plots above that pitch and roll profiles for the analytical solution and the computational Draper Laboratory solution have similar shape. The difference between roll profiles is less than between pitch profiles.

Figures 11, 12, and 13 provide comparisons of the torque reduction for the analytical and computational solutions for the 5400 second maneuver.

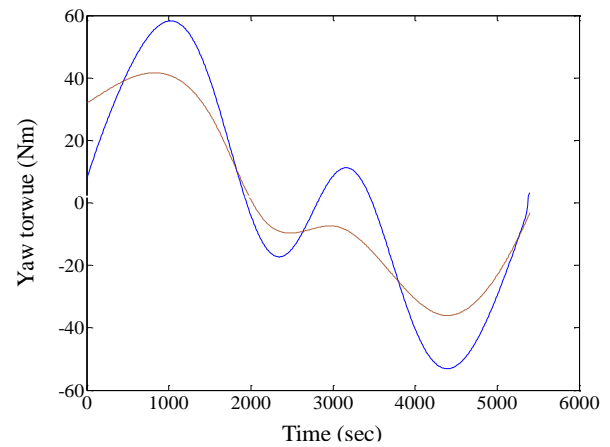


Figure 13. Yaw torque comparison for the 5400 second yaw maneuver.

It can be seen from the plots in Figures 11, 12, and 13 that for the 5400 second maneuver the torque reduction in roll for the analytical solution is similar to the Draper Laboratory solution. Also, for the analytical solution, the torque reduction is more for pitch and less for yaw.

Simulations using the ISS flight software were performed to confirm the validity and accuracy of the proposed analytical solution. The simulation results showed similar propellant consumption for both analytical and computational methods.

#### XIV. CONCLUSIONS

An analytical solution for optimizing the ISS yaw attitude maneuvers was suggested in this paper.

While approximate, the suggested solution provides optimization results that agree with the existing computational solution obtained by Draper Laboratory. The suggested analytical solution provides a new method for space vehicle maneuver optimization, which is automatic and less complicated.

#### XV. REFERENCES

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